Effiziente Algorithmen

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Assignment 3

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Exercise 3.1. (8) *Size of skip lists*

After inserting n elements, what is the expected space used by the skip list implementation described in the lecture notes? Observe that in the worst case the space consumption may grow indefinitely.

Exercise 3.2. (8) *Fingerprinting*

Let $a \neq b$ be two integers from the interval $[1, 2^n]$. Let p be a random prime with $1 \leq p \leq n^c$. Prove that $prob(a \equiv b \mod p) \leq \frac{c \cdot \ln(n)}{n^{c-1}}$.

Hint: As a consequence of the prime number theorem, exactly $\frac{n}{\ln(n)} \pm o(\frac{n}{\ln(n)})$ many numbers from $1, \ldots, n$ are prime.

Conclusion: We can compress n bits to $O(\log(n))$ bits and get a quite small false-positive rate.

Exercise 3.3. (8) *Randomized pattern matching*

We want to detect whether a binary pattern $\mathcal{P}$ of length m occurs in a binary text $\mathcal{T}$ of length n where $m < n$.

State an algorithm that runs in time $O(n)$ where we assume that arithmetic operations on $O(\log_2 n)$ bit numbers can be executed in constant time. The algorithm should accept with probability 1 whenever $\mathcal{P}$ is a substring of $\mathcal{T}$ and reject with probability at least $1 - \frac{1}{n}$ otherwise.

Hint: Use fingerprinting.