Effiziente Algorithmen

Summer term 2012

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Assignment 6

Take home: 05/21/2012
Submit: 05/**29**/2012

Note: Solutions may be submitted by email. Solutions submitted after the lecture will not be graded.

Exercise 6.1. (8) *Closest pair*

Show how to implement step (2b) of Algorithm 2.21 in expected time $O(|P_i|)$.

Hint: Use universal hashing and in particular apply Theorem 2.20.

Exercise 6.2. (8) *Streaming data*

We receive a stream of $n - 1$ pairwise different numbers from the set $\{1, \ldots, n\}$.

Show how to determine the missing number with an algorithm that reads the stream once and uses a memory of only $O(\log_2 n)$ bits.

Exercise 6.3. (8) *Clique sizes in random graphs*

We consider the family of random graphs $G(n, p)$ with n nodes. Each possible edge is independently inserted into $G(n, p)$ with probability p. Let X_k be the number of cliques of size k in $G(n, p)$.

- Prove that $E(X_k) = \binom{n}{k} \cdot p^{\binom{k}{2}}$.

Now let $p = \frac{1}{2}$, i.e. we consider the family $G(n, \frac{1}{2})$.

- For $n \to \infty$ show that $E(X_{\log_2 n}) \geq 1$.
- For $n \to \infty$ show that $E(X_{c \cdot \log_2 n}) \to 0$ for a constant $c > 1$ to be found by you.

Conclusion: Random graphs will contain only small cliques w.h.p.

Hint: $(\frac{n}{k})^k \leq \binom{n}{k} \leq (\frac{n \cdot e}{k})^k$.