

Assignment 8

Take home: 06/04/2012

Submit: 06/11/2012

Note: It is understood that all of your statements have to be proven correct.

Note: Solutions may be submitted by email. Solutions submitted after the lecture will not be graded.

Exercise 8.1. (8)

Stationary distribution

Consider the Markov chain on the state set $\{0, 1, \dots, n\}$. The transition probabilities are $P_{n \rightarrow 0} = P_{n \rightarrow n} = P_{i \rightarrow 0} = P_{i \rightarrow i+1} = \frac{1}{2}$ where $i < n$. State the chain's stationary distribution.

Exercise 8.2. (4+4)

Odyssey

Consider the Markov chain on the state set $\{0, 1, \dots, n\}$ where $n = 2k$. The transition probabilities for state i ($0 < i < n$) are $\frac{1}{2}$ to state $i + 1$ and $\frac{1}{2}$ to state $i - 1$. The chain has reflecting ends, i.e. $p_{0,1} = p_{n,n-1} = 1$.

- State the stationary distribution.
- What is the expected number of steps to reach state n starting from state 0?

Hint: Can you derive a recurrence equation for the expected number e_i of steps to reach state n from state i ?

Exercise 8.3. (8)

Randomized 2-SAT

Assume that the 2-SAT formula f on n variables has a solution. Consider the following algorithm.

```
compute a random variable assignment
while  $\exists$  unsatisfied clause
  select an unsatisfied clause  $c$  uniformly at random
  select one of  $c$ 's variables  $x$  uniformly at random
  invert  $x$ 
```

Analyze the expected number of iterations of the loop until a satisfying variable assignment is found.

Hint: Design a Markov chain to model the behaviour of the algorithm.

Conclusion: This randomized algorithm is not as fast as tailor-made (linear time) algorithms but it is very simple and generalizes to the case of 3-SAT formulae, as we will see later on.