Effiziente Algorithmen

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Assignment 9

Take home: 06/11/2012
Submit: 06/18/2012

Note: It is understood that all of your statements have to be proven correct.

Note: Solutions may be submitted by email. Solutions submitted after the lecture will not be graded.

Exercise 9.1. (4+4) *Modeling random processes*

Theo Retician always walks to work and then back home. He owns three umbrellas. He carries exactly one iff it is raining and iff he has access to one in the place where his walk starts. Let the rain probability p for each single walk be given.

- What is the probability π_i to start a walk in a location with i umbrellas, assuming the number of walks is unbounded?
- What p makes Theo get wet most often?

  Hint: Theo gets wet iff he is in a location with zero umbrellas and it rains subsequently.

Exercise 9.2. (3+5) *Ergodicity*

We know that ergodic Markov chains have exactly one stationary distribution. Construct a Markov chain that has more than one stationary distribution.

Show that a Markov chain with two different stationary distrubutions has indeed infinitely many stationary distributions.

Exercise 9.3. (1+7) *Random walks in undirected graphs*

Consider the graph L_n that consists of the complete graph K_n on n nodes, a path $v_1 - v_2 - \cdots - v_n$ and an edge $\{u, v_1\}$ for exactly one node $u \in V(K_n)$.

- Prove that the cover time for L_n is at most $O(n^3)$.
- Prove that the cover time for L_n is at least $\Omega(n^3)$.

  Hint: Investigate the expected time to reach v_n from an arbitrary node of $a \in V(K_n) - \{u\}$.