Effiziente Algorithmen

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Assignment 10

Take home: 06/18/2012
Submit: 06/25/2012

Note: It is understood that all of your statements have to be proven correct.

Note: Solutions may be submitted by email. Solutions submitted after the lecture will not be graded.

Exercise 10.1. (8) *Random walks in directed graphs*

Prove that the cover time for a directed graph G can be exponential in the size of G.

Exercise 10.2. (8) *Universal traversals*

Consider the class $\mathcal{G}(n, m)$ of undirected connected graphs on n nodes and m edges. A universal traversal is a sequence $t = t_1 \dots t_k \in \{1, \dots, n\}^k$, where $k = O(poly(n, m))$, such that for *any* $G \in \mathcal{G}(n, m)$ it holds that t visits all nodes when starting at an arbitrary node of G. The traversal rule for the current node v at step $i \geq 1$ is: hop to v's neighbor number $t_i \mod \deg(v)$.

Prove that universal traversals exist.

Hint: Show that $prob(t \text{ is universal}) > 0$ if t is picked uniformly at random from $\{1, \dots, n\}^k$, where you determine k. Think about concatenating random walks. Which length should each of them have? How many are needed?

Exercise 10.3. (8) *k-SAT*

We analyze a version of Algorithm 3.17 for which the loop in step (3) is iterated $6n$ times. In this setting the result of Lemma 3.18 can be shown analogously by means of the following statement:

Let a random walk start in state d. If the probability to walk left (i.e., to decrease the state number) is at least $\frac{2}{3}$, then the walk reaches state 0 within $6d$ steps with probability at least $\frac{1}{2}$.

Prove the statement.