Parallel and Distributed Algorithms

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Assignment 3

Issue: 02.11.2009
Due: 09.11.2009

Problem 3.4 provides eight *extra* credit points. Thus, there are 32 points achievable on this assignment, but only 24 points are accounted for as 100%.

3.1. Problem (8) *Butterfly networks*

Prove the following statements:

(a) The butterfly network B_d and the wrapped butterfly have the same number $d \cdot 2^{d+1}$ of edges and both have degree 4. B_d has $(d+1) \cdot 2^d$ nodes, the wrapped butterfly has $d \cdot 2^d$ nodes.

(b) The diameter of B_d is $2 \cdot d$ and decreases to $\lfloor \frac{3d}{2} \rfloor$ for the wrapped butterfly.

3.2. Problem (8) *Randomized routing on the hypercube*

Consider the randomized routing algorithm on the hypercube (Algorithm 2.19 on p. 50). Show that there is a packet $P_w \in \mathcal{P}$, that blocks P_v at most once.

3.3. Problem (8) *The Message Passing Interface*

Show how to construct paths $v \to v \oplus b$ in the d-dimensional hypercube for each $b \in \{0,1\}^d$, such that no edge is used by more than one path in the same direction.

(As a consequence several collective communication functions can be implemented on the hypercube without any congestion.)

3.4. Problem (8) *Independent Set*

Construct an independent set of maximum size for d–dimensional hypercube Q_d. An independet set is a subset $I \subseteq \{0,1\}^d$ of vertices such that no two vertices in I are connected by an edge.