Parallel and Distributed Algorithms

Winter 2009/2010

5

Issue: 16.11.2009 **Due: 23.11.2009**

Information

Solutions in english or german are fine.

5.1. Problem (8) *Inverting a matrix*

Let A be a non singular lower triangonal matrix. We know that the linear system $Ax = b$ can be solved in $O(\frac{n^2}{P})$ computing time and with communication time $O(n^2)$ using the off-diagonal decomposition (see theorem 4.3). **Show** how to compute A^{-1} in time $O(\frac{n^3}{P})$ and communication time $O(n^2)$.

5.2. Problem (8) *Interleaved row decomposition*

Let A be a lower triangonal matrix. Based on the rowwise decomposition we introduce the *interleaved* row decomposition: process i receives all rows with indices in the set $\{j \cdot p + i | j = 1 \dots n, i = 1 \dots p\}$. Hence process 1 gets row 1, row $p + 1$, row $2p + 1$ and so on.

Why is the interleaved row decomposition better than the rowwise decomposition when used for back substitution? In particular, why should we expect speedups by a constant factor?

5.3. Problem (8) *Jacobi Relaxation*

Assume that matrix A has a nonzero diagonal and let x be the unique solution of the linear system $A \cdot x = b$. **Show** that

$$x(t+1) - x = M \cdot (x(t) - x)$$

holds and consequently $x(t) - x = M^t \cdot (x(0) - x)$ follows for all t.

5.4. Problem (8) *Jacobi Relaxation*

Assume that matrix A has a nonzero diagonal and that the vectors $x(t)$, with

$$x_i(t) = \frac{1}{A[i,i]} \cdot \left(b_i - \sum_{j \neq i} A[i,j] \cdot x_j(t-1) \right),$$

converge to a vector x^*, in other words $\lim_{t \to \infty} x(t) = x^*$. **Show** that $A \cdot x^* = b$ holds.