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Book Reviews

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Extremal Combinatorics: with Applications in Computer Science by Stasys Jukna, Springer, 2001, xvii + 375 pp. £32.50; $49.95, ISBN 3–540–66313–4.

This book is an introduction to extremal combinatorics. Particular emphasis is placed on applications to computer science. It aims to be accessible to graduate students in mathematics and computer science.

The book is split into sections, each on a particular area or a particular method, and nearly always followed by some applications in computer science that use these ideas. A wide range of topics are covered. Some examples should give the flavour: Turán's theorem, Hall's theorem, sunflowers, intersecting families, antichains, blocking sets, designs, linear algebra arguments (including constructive lower bounds for Ramsey numbers), Hadamard matrices, probabilistic arguments (with many examples), entropy, derandomization, Ramsey's theorem, the Hales–Jewett theorem. The applications in computer science tend to be to do with lower bounds for complexity: small depth formulae, monotone circuits (including Razborov's lower bounds), multi-party communication complexity. There are also sections on the algorithmic aspects of some of the combinatorial results.

The author has covered a huge amount of ground in this book. He has clearly read very widely, and it shows. Each topic is covered in a way that takes the reader from the start right up to the most recent results in the area. It is particularly nice to see the computer science applications appearing in the same book as, and even right next to, the pieces of extremal combinatorics that are closest to them. One of the things this book will certainly achieve is to bring to the attention of combinatorialists the vast amount of computer science, especially on circuit complexity, that is so close to what they do that they really ought to know about it.

It is worth adding that the book is clearly a 'labour of love'. The author's enthusiasm for the subject shines through on page after page: it is hard not to feel his excitement as one reads what he has written.

Unfortunately, although at first sight the book seems very attractive, on closer examination a number of negative features of the book become apparent, which to my mind make it unsuitable for a non-expert who wishes to learn about the area. This is a shame, particularly in view of the author's great enthusiasm for the subject.

First of all, there is the emphasis on techniques rather than results. This leads to a picture of extremal combinatorics as a huge rag-bag of results with no connecting themes.

One gets no overview of what the subject is about, of what the few key results are, of how it all fits together. It also makes learning very difficult. For example, we learn about constructive lower bounds for Ramsey numbers in Chapter 14, way before we have our introduction to Ramsey theory (Chapter 27). The author has done this because he wants examples of the linear algebra method, but it is hard to see how a student could get much out of these constructive lower bounds before learning about Ramsey's theorem itself.

Another example of strange order, or lack of overview, occurs with probabilistic arguments. In Chapter 18 we meet the famous Erdős proof of the exponential lower bound for Ramsey numbers, where it is correctly stated that this is an excellent illustration of probabilistic ideas (the 'counting sieve'). Yet much earlier on, in Chapter 6, there is the Erdős proof that a family of less than 2^{k-1} k-sets has a 2-colouring – this is also a random argument, indeed of almost identical form, but no comment is made on it at all. Again, this is very unhelpful to the reader.

Then there are some curious omissions, of results and of points of view. Perhaps the most striking is that there is no mention of the Kruskal–Katona theorem. This is astonishing in a book on extremal combinatorics, as it is perhaps *the* most important and central result in extremal combinatorics. It sheds so much light on antichains (the LYM inequality), intersecting families (the Erdős–Ko–Rado theorem), and so on. This omission is completely incomprehensible. It is as though a book on extremal graph theory did not contain Turán's theorem.

Another remarkable omission is that of the geometric point of view. There is not one drawing of the discrete cube. Indeed, we are told that one can view set systems as blobs drawn around some points, as incidence matrices, or as incidence graphs, but we are not told that we may view them as subsets of the n-dimensional cube! This seems extraordinary, as it is surely the most important mental picture of all. There is also no picture or mention of the n-cube as a succession of 'layers' (the rth layer being all the r-sets). Even the discussion of the LYM inequality does not talk about fractions of layers, or anything like that. All this is almost guaranteeing that students will not be exposed to a vital way of thinking about set systems – one that is at the heart of extremal combinatorics.

There is also a lack of basic insight into the subject. For example, there is a chapter on the inclusion-exclusion principle. There are plenty of examples, but nowhere is it stated that the principle is often not that useful in estimating sizes, because of the fact that there may be both positive and negative terms that are large. This is very strange, in view of the fact that the book does have a lot of material later on about estimating things (such as material on random graphs). Again, this is not giving students a good feel for the subject.

As another example of this, in Chapter 11 we meet Kleitman's lemma (on the correlation between up-sets). It is just stated and proved: there is no mention of the word 'correlation', no discussion of whether the result is plausible or not, in fact no discussion at all.

The lack of discussion, and lack of guidance for the reader, comes in all the time. Results are presented (stated and proved) with no indication of whether they are easy or hard. To take an early example, Proposition 1.8 (about covering k-sets by l-sets) is a triviality, but written down here it looks as though it has an idea in it and needs some notation, that might frighten a beginning reader. Why is there no comment to the reader that the result is easy? As another example, the proof of Lemma 2.8 (on densities of rows and columns in matrices) looks quite short, but with some complicated lines: is this a trivial proof or a proof that has a clever idea in it? This is the kind of thing that a student (or any reader) needs to be told, but here there is no comment at all.

For yet another example, in Chapter 28 the notion of an abelian group is introduced, and then we go on to some zero-sum theorems for finite abelian groups. Yet nowhere is the reader told that actually abelian groups are not just abstract things but very tangible ones (namely direct sums of cyclic groups). How can a reader who is not told about direct sums of cyclic groups possibly get a proper grasp of the zero-sum theorems, or indeed any theorems, about finite abelian groups?

The above examples are just a small selection of the similar things one finds all over the book. Perhaps the overriding problem is that it seems that there has not been a reworking by the author of the results he presents: they are presented as a large cookbook of individual items (and often not well explained). To summarize, I would recommend this book for someone who knows about extremal combinatorics and wishes to learn about circuit complexity and the like, but I am afraid that I would not recommend it to any student, or to anyone wishing to learn about extremal combinatorics.

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Topics in Graph Automorphisms and Reconstruction by Josef Lauri and Raffaele Scapellato, Cambridge University Press, 2003, 172 pp. £47.50; \$65.00 HB, £16.95; \$23.00, PB ISBN 0521821517HB, 0521529034 PB

Josef Lauri and Raffaele Scapellato have written an introductory textbook on graph isomorphisms and reconstruction. The book contains eleven short chapters, seven concerning graph automorphisms and related topics and four on reconstruction problems.

The first few chapters concentrate on automorphisms and transitivity. An *automorphism* of a finite graph G is a bijection from $V(G)$ to $V(G)$ that preserves adjacency. The graph is *vertex transitive* if the group $\mathrm{Aut}(G)$ of its automorphisms acts transitively on $V(G)$ and *edge transitive* if the corresponding action on $E(G)$ is transitive. If $\mathrm{Aut}(G)$ acts transitively on the collection of ordered pairs $\langle x, y\rangle$ such that xy is an edge then the graph is said to be *arc transitive*. Lauri and Scapellato discuss these notions in detail, and prove a number of basic results. For instance, a graph that is vertex and edge transitive need not be arc transitive, although examples are surprisingly hard to come up with (one is given in the exercises).

More stringent symmetry conditions are also considered: a *t-arc* is a sequence $x_0 \cdots x_t$ of vertices that form a walk in which no two adjacent edges are the same. The graph G is *t-arc transitive* if its automorphism group acts transitively on the collection of t-arcs. Tutte showed that there are no finite cubic t-arc transitive graphs with $t > 5$, while Weiss showed that with larger vertex degrees there are no finite t-arc transitive graphs for $t > 7$ (although there is a 7-arc transitive graph of order 728). These results are discussed, but the proofs lie beyond the scope of the book.

There is a rather more detailed account of Cayley graphs. For a finite group Γ and a set S of generators with $S = S^{-1}$, the Cayley graph $\mathrm{Cay}(\Gamma, S)$ is the graph with vertex set

Γ and edges $\{xy : xy^{-1} \in S\}$. The left action of Γ on itself is clearly an automorphism of Cay(Γ, S), and there is a careful account of when all automorphisms arise in this way. (It is an amusing exercise to show that there is *some* graph G with Aut(G) isomorphic to Γ.)

The book touches on a number of related topics, such as the attractive conjectures of Lovász that every vertex transitive graph has a Hamilton path, and every Cayley graph has a Hamilton cycle (Babai [1] makes the opposite conjectures). There are also discussions of growth rates of groups, distance-regular and strongly regular graphs, as well as a rather abstract account of graph products. Several families of graphs are considered explicitly, such as generalized Petersen graphs, Kneser graphs and metacirculant graphs.

The last third of the book concerns reconstruction problems. The Reconstruction Conjecture of Kelly and Ulam asserts that every finite graph with at least 3 vertices is determined by the collection of subgraphs obtained by deleting one vertex of G. Harary's Edge Reconstruction Conjecture claims that it is possible to reconstruct every finite graph with at least 4 edges from the collection of subgraphs obtained by removing an edge. (Both collections are given with multiplicities, so may contain many copies of the same graph.) The Reconstruction Conjecture is now sixty years old, and the Edge Reconstruction Conjecture is forty, but progress towards resolving the two conjectures has been slow. Lauri and Scapellato prove a number of standard results on reconstruction, including Kocay's Lemma concerning coverings by subgraphs, and Tutte's theorem on the reconstruction of the characteristic and chromatic polynomials. The final chapter contains a discussion of recent work on more general reconstruction problems.

The book has a reasonable number of exercises, and would be easily accessible to a beginning research student. Most of the results presented in detail are fairly elementary: readers interested in graph symmetry will swiftly move on to more advanced books such as Biggs [4] or Godsil and Royle [5], and the wide-ranging survey article of Babai [1], while reconstruction is covered in greater depth by the surveys of Bondy [2], Bondy and Hemminger [3] and Nash-Williams [6]. However, Lauri and Scapellato's book provides an accessible and pleasant introduction to these areas.

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